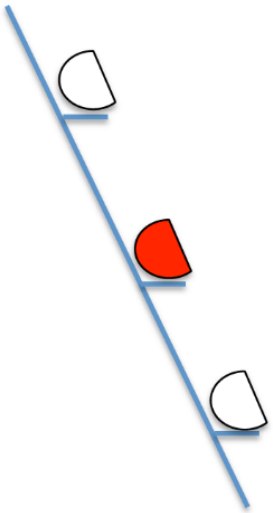
CS112 (Fall 2018) Homework 7
Epistemic Logic and Dynamic Epistemic Logic
Due December 7, 2018

1. Dynamic Epistemic Logic

   Three men are standing on a ladder, and each is wearing a hat. Each can see the color of the hats of the people below him, but not his own or those higher. It is common knowledge that only the colors red and white are possible, and that there are more white hats than red ones. We assume the following:

   - Agents = $\{B, M, T\}$, for Bottom, Middle, and Top;
   - The set of propositions corresponding to hats assigned to individuals is Props = $\{B_W, B_R, M_W, M_R, T_W, T_R\}$;
   - The worlds are Worlds = $\{B_W M_W T_W, B_R M_W T_W, ...\}$; and
   - The actual world is $B_W M_R T_W$.

   (a) Draw the epistemic model for this situation.

   (b) Who knows his color?

   (c) Now, the middle person says the following: "I don't know the color of my hat".

   i. Is this true?

   ii. Draw the update in the model.

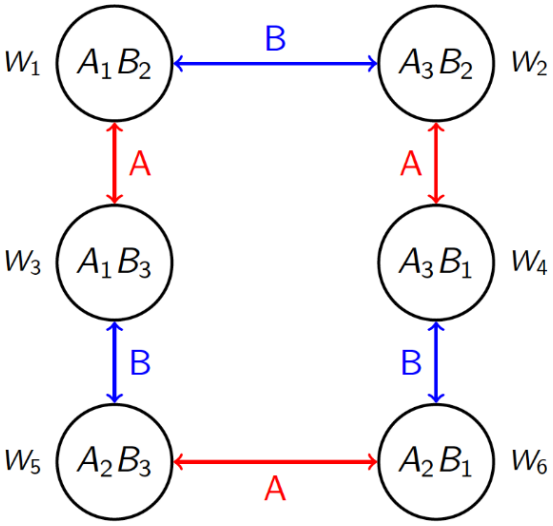
   iii. Who else knows his color now?

2. Epistemic Logic

(a) Two Agent Model

Consider the scenario with 3 cards: **1**, **2**, and **3**, and two players: *A*nn and *B*ob. Each of them is given one of the cards and the third is put back in the deck. Each of the players only knows his/her own card.

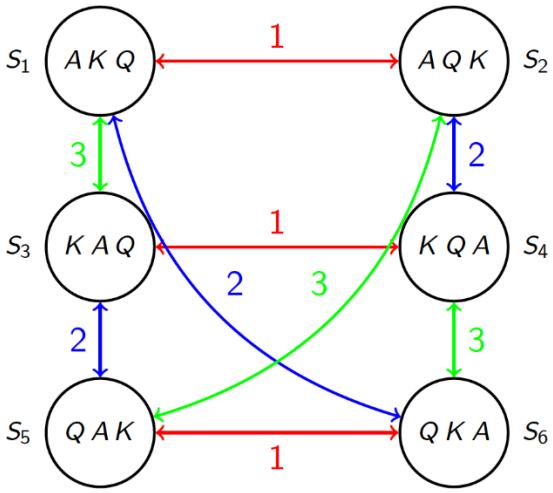
Here is the epistemic model $\mathcal{M}$, where the label A_iB_j means that Ann has card **i** and Bob has card **j**. An edge labeled with $X \in \{A, B\}$ indicates epistemic indistinguishability for player X. Determine whether each statement is true.



- $\mathcal{M}, w_1 \models K_A A_1 \wedge \neg K_B A_1$
- $\mathcal{M}, w_1 \models E_{A,B} \neg A_2$
- $\mathcal{M}, w_4 \models \neg K_A \neg B_3 \vee \neg K_B \neg A_1$
- $\mathcal{M}, w_1 \models D_{A,B}(\neg A_3 \wedge \neg B_3)$
- $\mathcal{M}, w_2 \models D_{A,B}(A_1 \wedge B_2)$
- $\mathcal{M}, w_1 \models K_A K_B \neg B_1$
- $\mathcal{M}, w_3 \models K_B K_A \neg(A_2 \wedge B_1)$
- $\mathcal{M}, w_3 \models K_B K_A K_B K_A \neg(A_2 \wedge B_1)$
- $\mathcal{M}, w_6 \models C_{A,B} \neg(A_3 \wedge B_3)$

(b) Three Agent Model

Consider the scenario with 3 cards: A, K, and Q, and 3 players: **1**, **2**, and **3**. Let $P_{i,X}$ mean that the player **i** holds the card X. Determine whether each statement is true.



- $\mathcal{M}, s_1 \models E_{\{1,2\}}(P_{1,A} \vee P_{3,A})$
- $\mathcal{M}, s_3 \models E_{\{1,3\}}(P_{2,A} \vee P_{1,K})$
- $\mathcal{M}, s_3 \models D_{\{1,2\}} P_{3,Q} \wedge D_{\{1,3\}} P_{2,A} \wedge D_{\{2,3\}} P_{1,K}$
- $\mathcal{M}, s_1 \models \neg C_{\{1,2\}} P_{1,A}$
- $\mathcal{M}, s_1 \models \neg C_{\{1,2\}}(P_{1,A} \vee P_{2,A})$
- $\mathcal{M}, s_1 \models \neg C_{\{1,2\}}(P_{1,A} \vee P_{2,A} \vee P_{3,A})$
- $\mathcal{M}, s_1 \models \neg C_{\{1,2\}}(P_{1,A} \vee P_{2,K})$