

CS112 (Fall 2018) Extra Credit Homework 6
Propositional Dynamic Logic
Due November 20, 2018

Note: This problem set is *optional*. If you do this problem set and do well on it, you will receive extra credit. If not, it will have no impact on your grade.

1. Recall that in dynamic logic, each program α is interpreted as a relation R_α , and operations on programs are interpreted as operations on relations (composition ; or \circ , union \cup , and iteration $*$). Below are some facts about these operations:

$$R^0 = I \text{ (where } I \text{ is the identity relation)}$$

$$R^{n+1} = R^n \circ R$$

$$Q \circ (R \circ S) = (Q \circ R) \circ S \text{ (composition of relations is associative)}$$

$$R \circ (\bigcup_i Q_i) = \bigcup_i (R \circ Q_i) \text{ (composition of relations distributes over union)}$$

$$(\bigcup_i Q_i) \circ R = \bigcup_i (Q_i \circ R) \text{ (ditto)}$$

$$R^* = I \cup R \cup (R \circ R) \cup \dots \cup R^n \cup \dots = \bigcup_{n \geq 0} R^n$$

$$I \circ R = R = R \circ I$$

Show the following:

(a) $R^n \circ R^m = R^{n+m}$ for $n, m \geq 0$ (hint: use induction on m)

(b) $R \circ R^* = R^* \circ R$

(c) $R \circ (Q \circ R)^* = (R \circ Q)^* \circ R$

(hint: use induction to show that $R \circ (Q \circ R)^n = (R \circ Q)^n \circ R$ for $n \geq 0$)

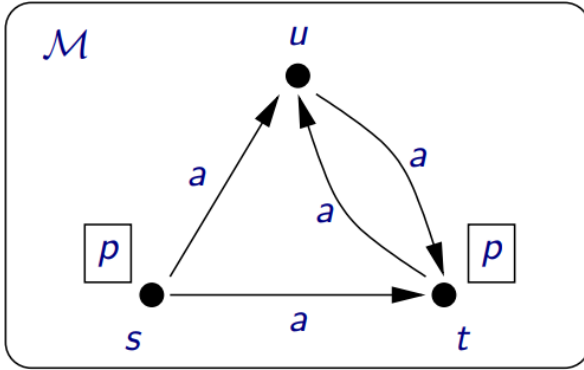
(d) $R^* = I \cup (R \circ R^*)$

2. Encode the following two programs in PDL:

while ϕ_1 do α ;
while ϕ_2 do β

if ϕ_1 then α ;
while $\phi_1 \vee \phi_2$ do
 if ϕ_2 then β else α

3. Let \mathcal{M} be the following model:



Let $\alpha = \text{if } p \text{ then } aa \text{ else } a$.

- (a) Does the formula $\langle \alpha \rangle p$ hold throughout $\widehat{\mathcal{M}}$? Justify your answer.
 - (b) Does the formula $[\alpha]p$ hold throughout $\widehat{\mathcal{M}}$? Justify your answer.
4. For the following pair of formulas, find a model and a world in it which distinguishes them:
 $[(\alpha \cup \beta)^*]p$ and $[\alpha^*]p \wedge [\beta^*]p$