— Advanced Logic —

Linear Temporal Logic
Computation Tree Logic

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Overview

Linear temporal logic (LTL):

- describes properties of paths (individual executions)
- no modalities to reason about branching

Computation tree logic (CTL):

- is a branching-time logic
- time has a tree structure (multiple possible futures)
- has modalities for reasoning about the branching structure

Linear Temporal Logic (LTL)

Linear temporal logic (LTL) is defined by:

$$\phi ::= p \mid \top \mid \neg\phi \mid \phi \wedge \phi \mid \phi \mathsf{U} \phi \mid \mathsf{X}\, \phi$$

where $p \in \Omega$

LTL formulas have meaning on individual computation paths:

- let $\pi = s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$ a path; write π^i for $s_i \rightarrow s_{i+1} \rightarrow \ldots$

The path π satisfies ϕ, $\pi \models \phi$, is defined by:

1. $\pi \models p$ iff $s_1 \in V(p)$
2. $\pi \models \top$; $\pi \models \neg\phi$ iff $\pi \not\models \phi$; $\pi \models \phi_1 \wedge \phi_2$ iff $\pi \models \phi_1$ and $\pi \models \phi_2$

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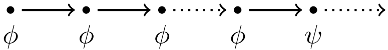
3. $\pi \models \phi \mathsf{U} \psi$ (ϕ is true until ψ is true)



formally: for some $i \geq 1$, $\pi^i \models \psi$ and for all $j < i$, $\pi^j \models \phi$

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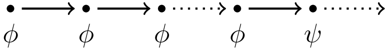
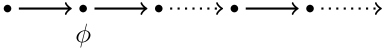


formally: for some $i \geq 1$, $\pi^i \models \psi$ and for all $j < i$, $\pi^j \models \phi$

4. $\pi \models \mathsf{X}\, \phi$ (ϕ is true in the next moment in time)



formally: $\pi^2 \models \phi$

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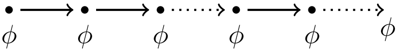

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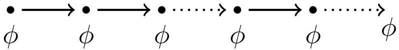

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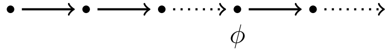

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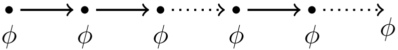

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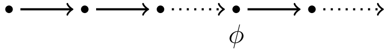

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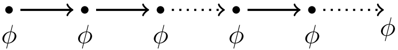

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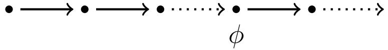

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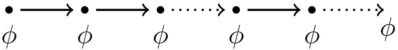

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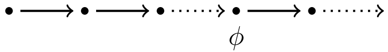

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Binding strength: $\neg$, X , F , G stronger than U than $\wedge, \vee$ than $\rightarrow, \leftrightarrow$

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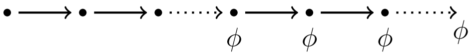

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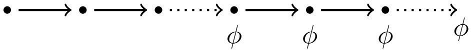

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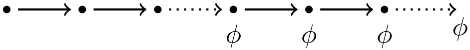
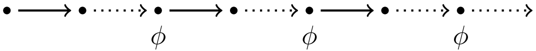


- G F ϕ : always eventually ϕ (in every suffix, at some point ϕ holds)



LTL: Models

$\mathfrak{M}, s \models \phi$ if ϕ is satisfied on every path starting at s.
$\mathfrak{M} \models \phi$ if ϕ is satisfied on every path starting from the initial state.

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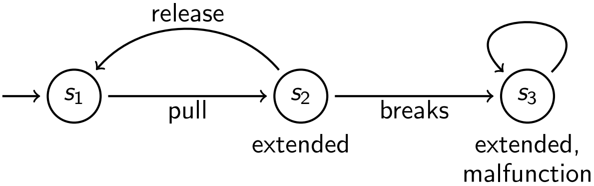

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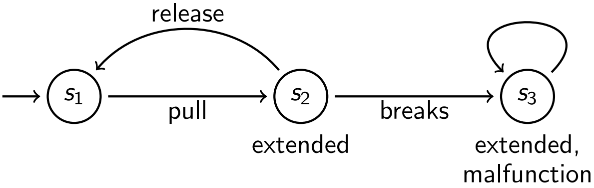

Which of the states satisfies the following?

$? \models \mathrm{X}$ extended

$? \models \mathrm{X}\ \mathrm{X}$ extended

$? \models \mathrm{F}$ extended

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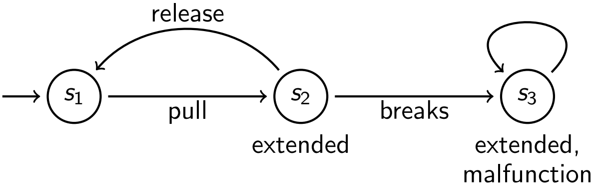

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? $\models$ G (¬extended → X extended)
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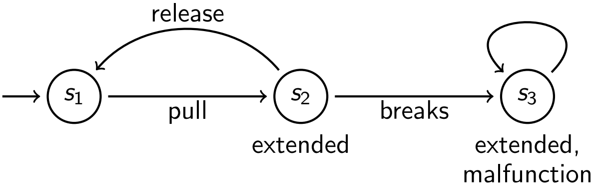

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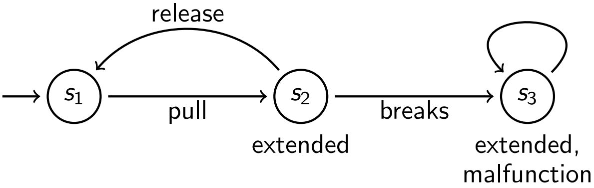

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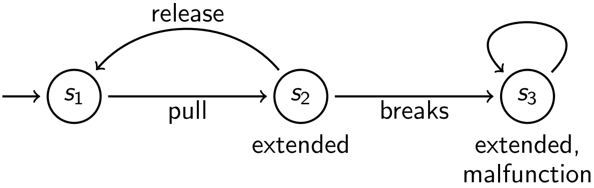

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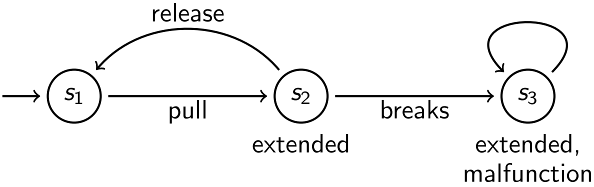

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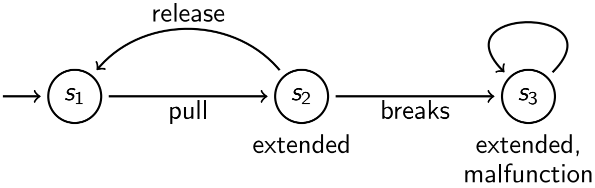

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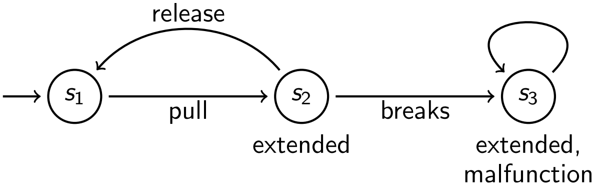

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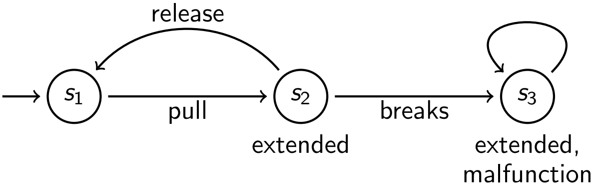

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$\mathfrak{M}, s_1, s_2, s_3 \models \mathsf{G}\ \mathsf{F}\ \text{extended}$

$\mathfrak{M}, s_3 \models \mathsf{F}\ \mathsf{G}\ \text{extended}$

$\mathfrak{M}, s_1, s_2, s_3 \not\models \neg\, \mathsf{F}\ \mathsf{G}\ \text{extended}$

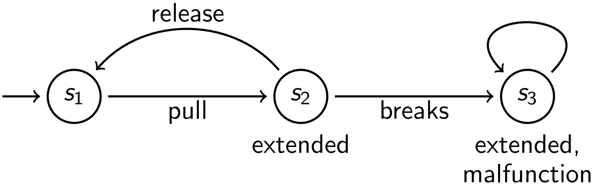
$? \models \mathsf{G}\ (\neg\text{extended} \to \mathsf{X}\ \text{extended})$

$? \models \mathsf{G}\ (\text{extended} \to \mathsf{X}\ \neg\text{extended})$

Note that: $\mathfrak{M} \not\models \mathsf{F}\ \mathsf{G}\ \text{extended}$ and $\mathfrak{M} \not\models \neg\, \mathsf{F}\ \mathsf{G}\ \text{extended}$!

LTL: Models

$\mathfrak{M}, s \models \phi$ if ϕ is satisfied on every path starting at s.
$\mathfrak{M} \models \phi$ if ϕ is satisfied on every path starting from the initial state.



Which of the states satisfies the following?

$\mathfrak{M}, s_1, s_3 \models$ X extended

$\mathfrak{M}, s_2, s_3 \models$ X X extended

$\mathfrak{M}, s_1, s_2, s_3 \models$ F extended

$\mathfrak{M}, s_3 \models$ G extended

$\mathfrak{M}, s_1, s_2, s_3 \models$ G F extended

$\mathfrak{M}, s_3 \models$ F G extended

$\mathfrak{M}, s_1, s_2, s_3 \not\models \neg$ F G extended

$\mathfrak{M} \models$ G ($\neg$extended $\rightarrow$ X extended)

? $\models$ G (extended $\rightarrow$ X $\neg$extended)

Note that: $\mathfrak{M} \not\models$ F G extended and $\mathfrak{M} \not\models \neg$ F G extended !

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$\mathfrak{M}, s \models \phi$ if ϕ is satisfied on every path starting at s.
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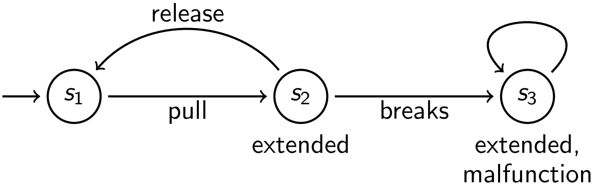

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$$\mathfrak{M}, s_2, s_3 \models \text{X X extended}$$

$$\mathfrak{M}, s_1, s_2, s_3 \models \text{F extended}$$

$$\mathfrak{M}, s_3 \models \text{G extended}$$

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$$\mathfrak{M}, s_1, s_2, s_3 \not\models \text{G } (\text{extended} \rightarrow \text{X } \neg\text{extended})$$

Note that: $\mathfrak{M} \not\models \text{F G extended}$ and $\mathfrak{M} \not\models \neg\,\text{F G extended}$!

LTL: Equivalence of Formulas

LTL formulas ϕ and ψ are semantically equivalent, denoted by $\phi \equiv \psi$, if they are true for the same paths

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$$\mathsf{X}\ (\phi \vee \psi) \equiv \mathsf{X}\ \phi \vee \mathsf{X}\ \psi$$

$$\mathsf{X}\ (\phi \wedge \psi) \equiv \mathsf{X}\ \phi \wedge \mathsf{X}\ \psi$$

$$\mathsf{F}\ (\phi \wedge \psi) \equiv \mathsf{F}\ \phi \wedge \mathsf{F}\ \psi$$

$$\mathsf{F}\ (\phi \vee \psi) \equiv \mathsf{F}\ \phi \vee \mathsf{F}\ \psi$$

$$\mathsf{G}\ (\phi \wedge \psi) \equiv \mathsf{G}\ \phi \wedge \mathsf{F}\ \psi$$

$$\mathsf{G}\ (\phi \vee \psi) \equiv \mathsf{G}\ \phi \vee \mathsf{F}\ \psi$$

$$\rho\ \mathsf{U}\ (\phi \vee \psi) \equiv (\rho\ \mathsf{U}\ \phi) \vee (\rho\ \mathsf{U}\ \psi)$$

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$$\mathsf{F}\ \phi \equiv \phi \vee \mathsf{X}\ (\mathsf{F}\ \phi)$$

$$\mathsf{G}\ \phi \equiv \phi \wedge \mathsf{X}\ (\mathsf{G}\ \phi)$$

$$\phi\ \mathsf{U}\ \psi \equiv \phi\ \mathsf{U}\ (\phi\ \mathsf{U}\ \psi)$$

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$\mathsf{F}\ (\phi \vee \psi) \equiv \mathsf{F}\ \phi \vee \mathsf{F}\ \psi$

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$\mathsf{F}\,\mathsf{F}\,\phi \equiv \mathsf{F}\,\phi$

$\mathsf{G}\,\mathsf{G}\,\phi \equiv \mathsf{G}\,\phi$

$\mathsf{F}\,\mathsf{G}\,\phi \equiv \mathsf{G}\,\mathsf{F}\,\phi$

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$\neg\mathsf{G}\,\phi \equiv \mathsf{F}\,\neg\phi$

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$\mathsf{X}\ (\phi \land \psi) \equiv \mathsf{X}\ \phi \land \mathsf{X}\ \psi$

~~$\mathsf{F}\ (\phi \land \psi) \equiv \mathsf{F}\ \phi \land \mathsf{F}\ \psi$~~

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$\neg \mathsf{F}\ \phi \equiv \mathsf{G}\ \neg \phi$

$\neg \mathsf{G}\ \phi \equiv \mathsf{F}\ \neg \phi$

$\mathsf{F}\ \phi \equiv \phi \lor \mathsf{X}\ (\mathsf{F}\ \phi)$

$\mathsf{G}\ \phi \equiv \phi \land \mathsf{X}\ (\mathsf{G}\ \phi)$

$\phi\ \mathsf{U}\ \psi \equiv \phi\ \mathsf{U}\ (\phi\ \mathsf{U}\ \psi)$

Mutual Exclusion

- multiple processes
- a shared resource that can only be used by one process at a time

process Q

shared resource

process P

Mutual Exclusion

- multiple processes
- a shared resource that can only be used by one process at a time

Q

non critical

...

C_Q critical section

...

non critical

P

non critical

...

C_P critical section

...

non critical

To solve conflicts: processes agree on a negotiation protocol.

- mutual exclusion: never more than one process in the critical section

Mutual Exclusion

- multiple processes
- a shared resource that can only be used by one process at a time

Q
non critical
...
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...
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P
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non critical

To solve conflicts: processes agree on a negotiation protocol.

- mutual exclusion: never more than one process in the critical section

$$\mathsf{G}\ \neg(C_Q \wedge C_P)$$

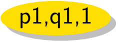
Mutual Exclusion: Attempt 1

- boolean variable *free* = 1

```
P
loop forever
  p1: wait for $free = 1$
  p2: $free = 0$
  $C_P$: critical section
  p4: $free = 1$
```

```
Q
loop forever
  q1: wait for $free = 1$
  q2: $free = 0$
  $C_Q$: critical section
  q4: $free = 1$
```

For such a program we compute the state space:



p1,q1,1

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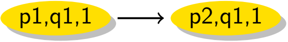

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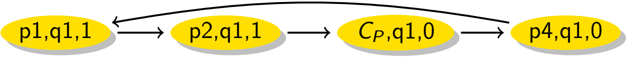

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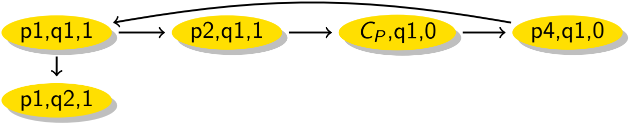

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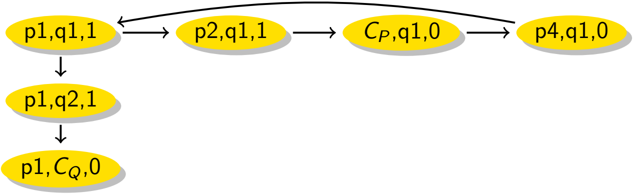

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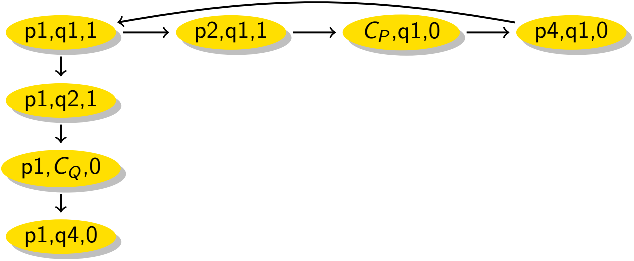

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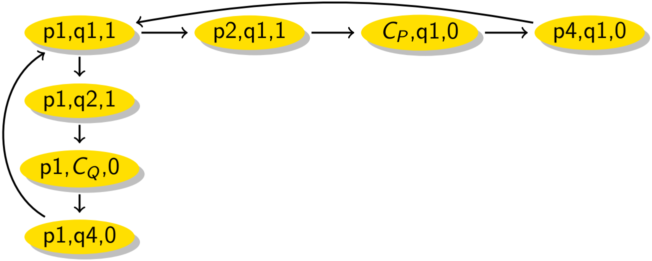

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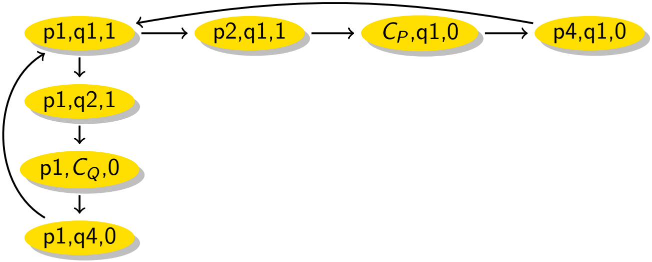

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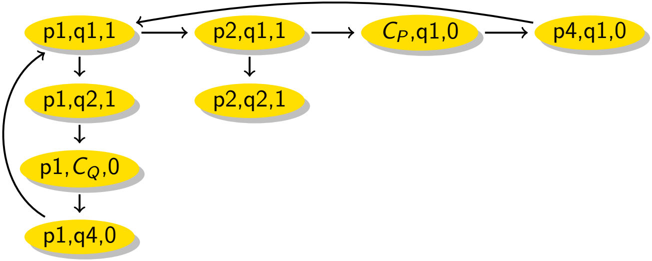

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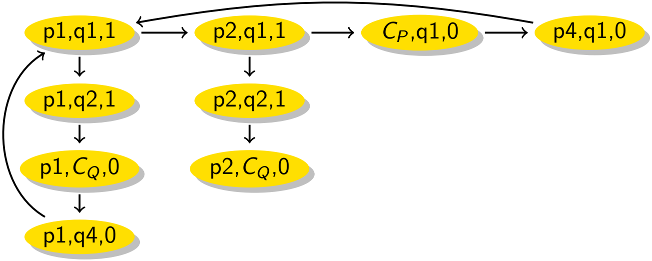

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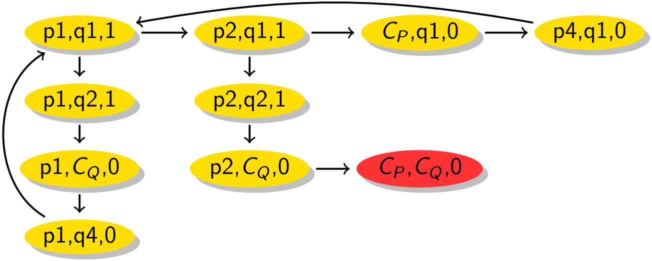

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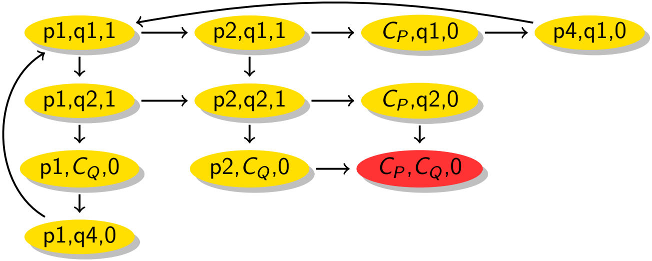

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P	*Q*
loop forever p1: wait for $free = 1$ p2: $free = 0$ C_P: critical section p4: $free = 1$	loop forever q1: wait for $free = 1$ q2: $free = 0$ C_Q: critical section q4: $free = 1$

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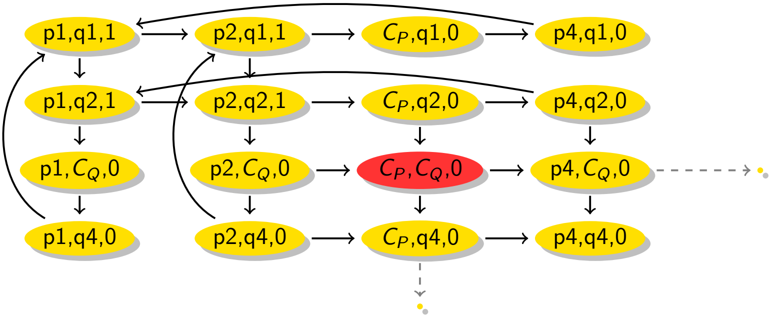
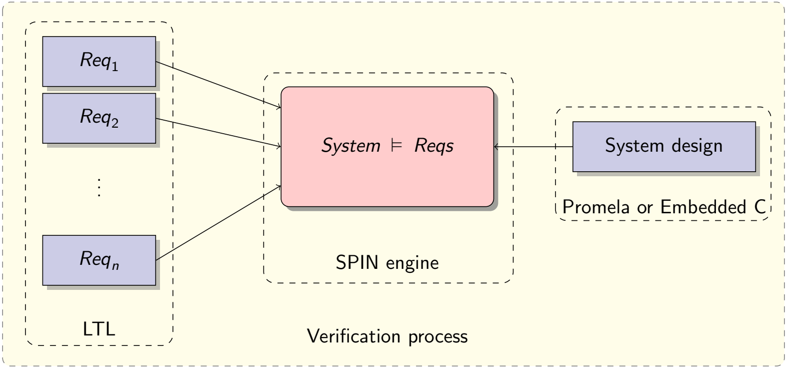


Model Checking

1. Formalize the system design
2. Formalize the validation requirements
3. Validate: system meets requirements



Req_1

Req_2

⋮

Req_n

LTL

$System \models Reqs$

SPIN engine

System design

Promela or Embedded C

Verification process

Mutual Exclusion: Peterson

- boolean variables $x = 0$, $y = 0$, $t = 0$

P

```
loop forever
  p1: $x = 1$
  p2: $turn = 1$
  p3: wait for $y = 0$ or $t = 0$
  $C_P$: critical section
  p4: $x = 0$
```

Q

```
loop forever
  q1: $y = 1$
  q2: $turn = 0$
  q3: wait for $x = 0$ or $t = 1$
  $C_Q$: critical section
  q4: $y = 0$
```

LTL: Applications

Safety properties

- “nothing bad ever happens”

$$\mathsf{G}\ \neg(\text{reactor temperature} > 1000)$$

- invariant: “*a* is always false”

Liveness properties

- “something good will eventually happen”

$$\mathsf{G}\ (\text{ordered} \rightarrow \mathsf{F}\ \text{delivered})$$

- termination: “the system will eventually terminate”
- response: “if action *a* occurs then *b* eventually will occur”

Deadlock freeness

- deadlock state: “a state where no actions are possible”
- no deadlocks: there is always some next state

$$\mathsf{G}\ (\neg\text{terminated} \rightarrow \mathsf{X}\ \top)$$

Industrial Case Studies I

Figure: After Flood Disaster (1953), Maeslant Barrier (Maeslantkering)

Industrial Case Studies: Flood Control

Verification of the interface between BOS and BESW:

- Beslis- en Ondersteunend Systeem (BOS)
- BEsturingsSysteem Waterweg (BESW)
- BOS takes the decision to move the barrier
- BESW performs this task

Even deadlocks were found in BESW!

Industrial Case Studies II

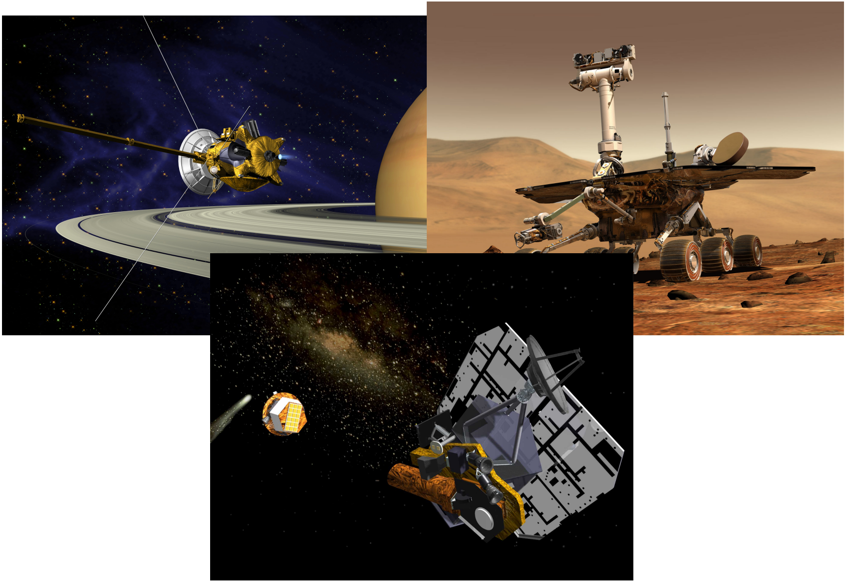
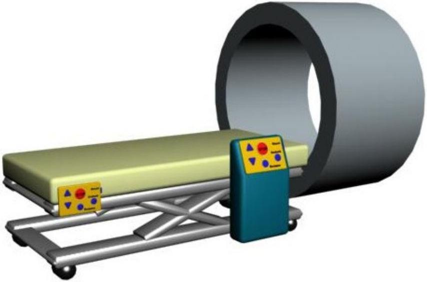
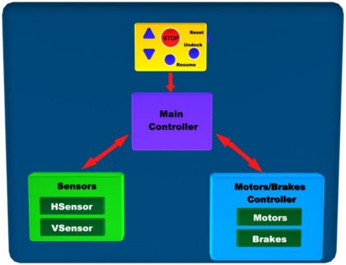


Figure: NASA Mission Critical Software: Cassini, Mars Rovers, Deep Impact

Industrial Case Studies III



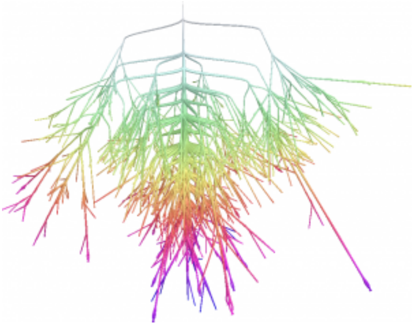


Main Controller

Sensors
- HSensor
- VSensor

Motors/Brakes Controller
- Motors
- Brakes

State Space Explosion



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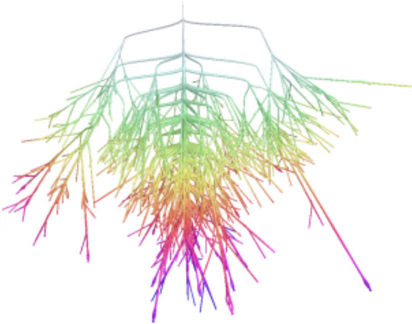
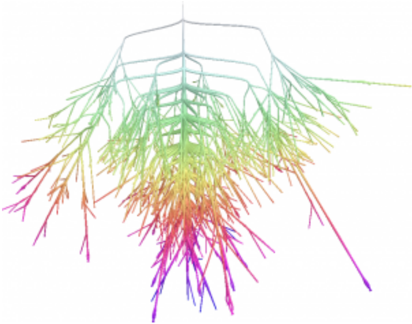


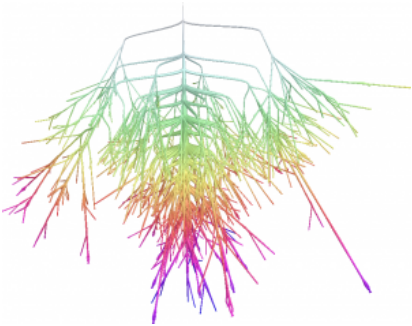
- Assume A_1, A_2, $\ldots$ are a processes each having 10 states

State Space Explosion



- Assume A_1, A_2, . . . are a processes each having 10 states
- Then A_1 and A_2 together have 100 states.

State Space Explosion



- Assume A_1, A_2, . . . are a processes each having 10 states
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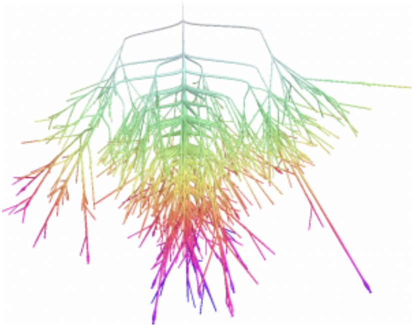
- Then A_1, . . . , A_n together have 10^n states.

State Space Explosion



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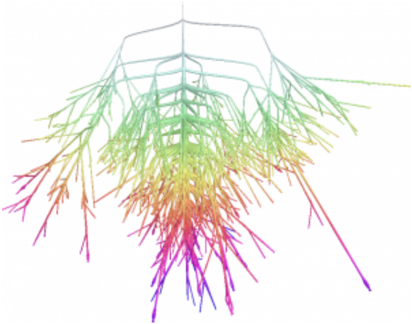

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Computation Tree Logic (CTL)

Computation Tree Logic (CTL) is defined by:

$$\phi ::= p \mid \top \mid \neg\phi \mid \phi \wedge \phi \mid \phi \text{ EU } \phi \mid \text{EG } \phi \mid \text{EX } \phi$$

where $p \in \Omega$

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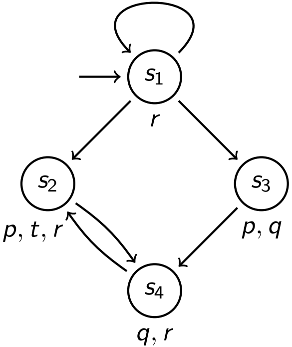
$$\phi \text{ AU } \psi = \neg(\neg\psi \text{ EU } (\neg\phi \wedge \neg\psi)) \wedge \neg\text{EG } \neg\psi$$

CTL: Examples



Which of the states satisfies the following?

$? \models \mathrm{AF}\ t$

$? \models \neg\mathrm{EG}\ r$

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$? \models \mathrm{EX}\ q$

$? \models \mathrm{AX}\ q$

$? \models \mathrm{EF}\ q$

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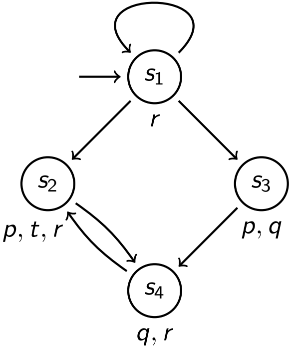
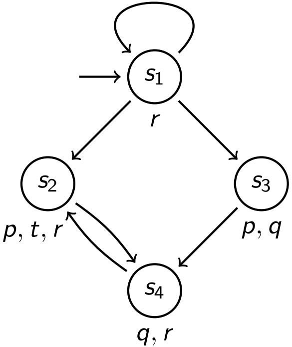


Which of the states satisfies the following?

$$
\begin{aligned}
\mathfrak{M}, s_2, s_3, s_4 &\models \text{AF } t \\
? &\models \neg\text{EG } r \\
? &\models t \text{ EU } q \\
? &\models \text{EX } q \\
? &\models \text{AX } q \\
? &\models \text{EF } q
\end{aligned}
$$

CTL: Examples



Which of the states satisfies the following?

$$\mathfrak{M}, s_2, s_3, s_4 \models \text{AF } t$$

$$\mathfrak{M}, s_3 \models \neg\text{EG } r$$

$$? \models t \text{ EU } q$$

$$? \models \text{EX } q$$

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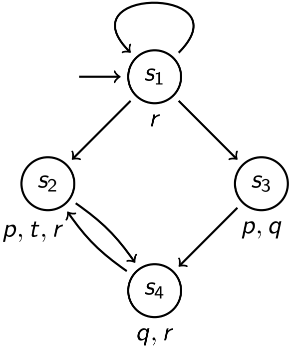
CTL: Examples



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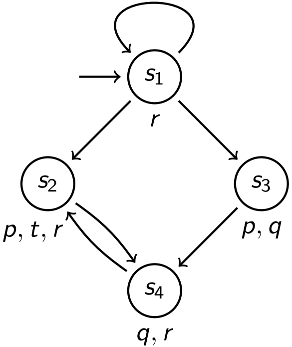

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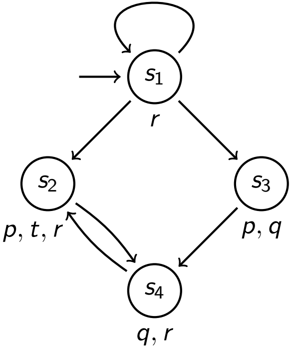

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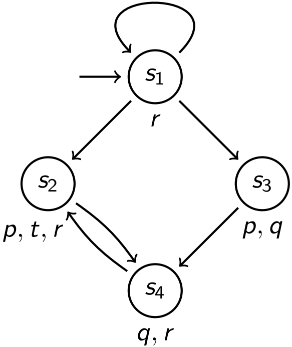
CTL: Examples



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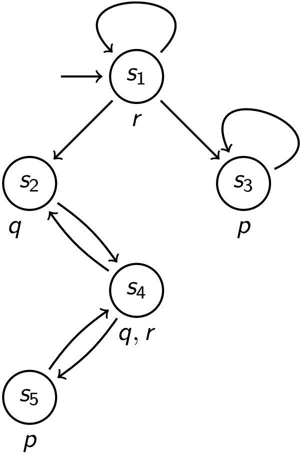
CTL: Examples



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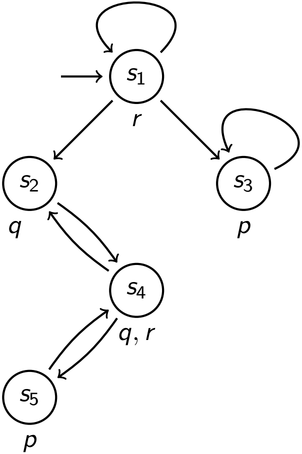
CTL: Examples



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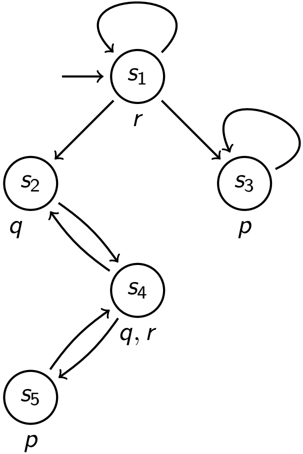
CTL: Examples



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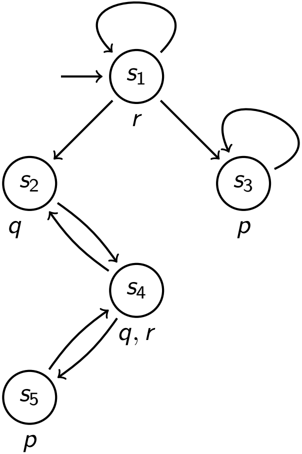
CTL: Examples



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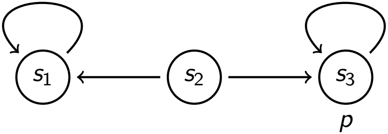
CTL vs LTL

CTL vs LTL

- a CTL formula necessitating E cannot be expressed in LTL

$$\mathrm{EX}\ p$$



CTL vs LTL

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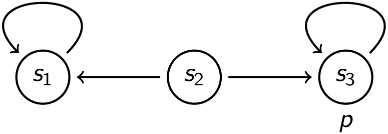

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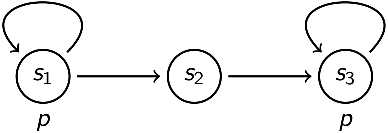

CTL vs LTL

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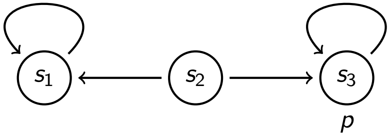

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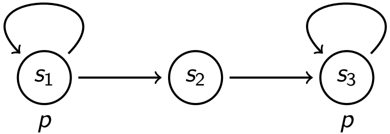

- the LTL formula G F $p \rightarrow$ F q cannot be expressed in CTL